ISSN: 0976 - 2965
IJBET (2010), 1(1):37-42

A Mathematical Model for Modified Herschel-Bulkley Fluid in Modeled Stenosed Artery under the Effect of Magnetic Field

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ABSTRACT:
The study focuses on the behavior of blood flow through an axially non-symmetric but radially symmetric stenosed artery when blood is represented as Herschel-Bulkley fluid and a uniform magnetic field is applied on the flow. The effect of magnetic field is considered in the transverse direction of blood flow and viscosity of blood is taken as radial co-ordinate dependent. The expression for velocity, resistance to flow and wall shear stress is derived. It is observed that the magnitudes of the blood flow characteristics significantly increase with in the red cell concentration, which is depending on hematocrit value of blood. The importance of the decreasing velocity and resistance with increasing Hartmann numbers and stenosis shape parameter is also pointed out. Resistance to flow and velocity reduces the abnormalities of the artery in the presence of magnetic field.

Keywords: Resistance to flow, Shape of stenosis, Magnetic field, Wall shear stress, Viscosity of blood, Herschel-Bulkley fluid.

INTRODUCTION

The idea of using magnetic field for therapeutic reasons is almost as old as the discovery of magnetism. It is well known that the external magnetic field has considerable effect on the biological system of human. The magnetic behaviour of blood is justified due to the haemoglobin molecule, a form of ironoxides, which is present at a uniquely high concentration in the mature red blood cells [1]. It is found that the blood possess the property of diamagnetic material when oxygenated and paramagnetic when deoxygenated. In an arterial constriction blood viscosity increases due to the conservation of mass, producing increased wall shear stress in the region of the blood acceleration. Theoretical and experimental studies of the circulator disorders, know to be responsible for deaths in most of the cases, have been the subject of scientist research. With the advent of the discovery that the cardiovascular disease (stenosis or arteriosclerosis) is closely associated with the flow conditions in the blood vessels, scientists have been focusing attention on this area of biomechanics. The medical term "stenosis" means narrowing of any body passage, tube or orifice, is the abnormal and unnatural growth in arterial wall thickness that develops at the various locations of the cardiovascular system under diseased conditions and can result in serious consequences by reducing or occluding the blood supply. It has been suggested that the deposits of the cholesterol and proliferation of connective tissue from plaques that enlarge and restrict the blood flow. If the plaques are present in an artery, normal blood flow is disturbed. One may expect that if such an event occurs, the flow characteristics in the vicinity of the resulting protuberance may be significantly altered. Several attempts have been made in the literature to study the effect of stenosis on the blood flow characteristics, including the important contribution of [2-5]. The effect of stenosis on the resistance to flow through artery by considering the behaviour of blood as a Herschel-Bulkley fluid model has been studied by [6-8]. The response of blood flow through an artery under stenotic conditions has been attempted by [9, 10]. But the problem of flow under magnetic effects becomes much more difficult through arery with stenosis at some region and very few researchers have attempted the flow problem under magnetic effects.

A little attention has been made [11-13] to study the effect of magnetic field on physiological fluid flows. It has been found that with the help of magnets, the flow of blood in arteries is properly regulated with the regulation in flow. It has also been reported by [14, 15, and 16] that the biological systems are affected by magnetic field. Amos [17] concluded a similar study in which they obtained numerical results for the stream function and vorticty using the Gelarkin technique of the finite element method. The effect of an externally applied magnetic field over the flow characteristics of blood in a single stenosed artery has been analysed by [18]. Computational modeling of flow in diseased arteries using realistic geometries derived from magnetic resonance imaging is gaining favour as a tool for understanding and predicting cardiovascular disease [19, 20]. This is because in vivo measurements of the flow field in an artery can be costly and are only possible for arteries that are easily accessible. Such measurement also provides very limited details about the flow. Numerical models of arterial flow may provide research with a platform for testing how flow conditions can be modified in order to change or prevent disease. The published literature on the stenosis further reveals that very few studies are concerned with the problem of symmetric stenosis. In an actual situation, however, the increase in the arterial wall thickness would not be symmetrical. To generalize the problem further, an attempt is therefore made in the present investigation. In this study a

mathematical model is proposed to describe blood flow through an axially non-symmetric but radially symmetric stenosed artery when blood is represented as Herschel-Bulkley fluid and a uniform magnetic field is applied on the flow.

FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of blood in a uniform circular tube with an axially non-symmetric but radially symmetric mild stenosis. The geometry of the wall surface can be described as [Fig.1],

$$\begin{aligned} R'(z) &= R_0[1 - A[L_0'^{(m-1)}(z'-d') - (z'-d')^m], && d' \le z' \le d' + L_0' \\ &= R_0, && \text{otherwise,} \end{aligned} \tag{1}$$

where $R'(z)$ is the radius of the artery with stenosis, R_0 is the constant radius of the artery. L_0 is the stenosis length and d indicates the stenosis location, and $m \ge 2$ is a parameter determining the stenosis shape and is referred to as stenosis shape parameter. Here axially symmetric stenosis occurs when m = 2, the parameter A is given by:

$$A = \frac{\delta_S}{R_0 (L_0')^m} \frac{m^{m/(m-1)}}{(m-1)},$$

where δs denotes the maximum height of stenosis at $z' = d' + L_0' / m^{1/(m-1)}$. The ratio of the stenosis height to the radius of the normal artery is much less than unity.

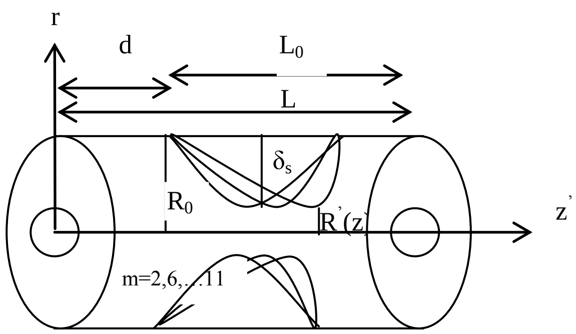

Fig (1) Geometry of Stenosis

Herschel-Bulkley fluid model

The stress-strain relation of Herschel-Bulkley fluid is given as:

$$\begin{aligned} f(\tau') &= \left(-\frac{du'}{dr'}\right) = \frac{1}{\mu'}\left(\tau' - \tau'_0\right)^n, && \tau' \ge \tau'_0 \\ f(\tau') &= \left(-\frac{du'}{dr'}\right) = 0, && \tau' \le \tau'_0 \end{aligned} \tag{2}$$

where $\tau = \left(-\frac{dP'}{dz'}\frac{r'}{2}\right)$, $\tau_0 = \left(-\frac{dP'}{dz'}\frac{R'_c}{2}\right)$,

and μ denotes Herschel-Bulkley viscosity coefficient, τ_o is yield stress, τ is shear stress, R_c is the radius of the plug-flow region, u is the axial velocity along the z direction and n is the flow behavior index. The relation correspond to the vanishing of the velocity gradients in regions, in which the shear stress τ is less than the yield stress τ_o this implies a plug flow wherever $\tau \le \tau_o$ when the shear rates in the fluid are very high, $\tau \ge \tau_o$, the power-law fluid behavior is indicated.

Governing equations

Governing equation can be written as:

$$\left(-\frac{\partial P'}{\partial Z'}\right) + \frac{1}{r'}\frac{\partial}{\partial r'}\left(\mu' r' \frac{\partial u'}{\partial r'}\right) + \left(J' \times B'\right) = 0 \tag{3}$$

$$J' = \sigma(E' + u' \times B') \text{ and } \mu' = \mu_0 \left(\frac{r'}{R_0}\right)^{(-M)} \tag{4}$$

Where E' is electric field, B' is magnetic field, σ is electric conductivity, J' is magnetic flux and M is a parameter depending upon the hematocrit value.

Boundary conditions

Following boundary conditions are introduced to solve the above equations:

$$\begin{aligned} \left(\frac{\partial u'}{\partial r'}\right) &= 0 && \text{at} \quad r' = 0 \\ u' &= 0 && \text{at} \quad r' = R(z) \end{aligned} \tag{5}$$

Non dimensional scheme

$$R = \left(\frac{R'}{R_0}\right), \mu = \left(\frac{\mu'}{\mu_0}\right), r = \left(\frac{r'}{R_0}\right), L_0 = \left(\frac{L_0'}{L}\right), \sigma = \left(\frac{\sigma'}{R_0}\right), \tag{6}$$

$$Re = \left(\frac{\rho U_0 R_0}{\mu_0}\right), d = \left(\frac{d'}{L}\right), z = \left(\frac{z'}{L}\right), P = \left(\frac{P'}{\rho U_0^2}\right), u = \left(\frac{u'}{U_0}\right)$$

The governing equations and boundary conditions are transformed to:

$$\begin{aligned} R(z) &= 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], && d \le z \le d + L_0 \\ &= 1, && \text{otherwise,} \end{aligned} \tag{7}$$

Where, $A = \frac{\delta}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)}$

$$r^{-M}\left(\frac{\partial^2 u}{\partial r^2}\right) + (1-M) r^{-(1+M)}\left(\frac{\partial u}{\partial r}\right) - H^2 u = Re\, \varepsilon \left(\frac{\partial p}{\partial z}\right) \tag{8}$$

Where, $J = \sigma(E + u \times B)$, $\mu = r^{-M}$, and $H^2 = \left(\frac{B_0^2 R_0^2 \sigma}{\mu_0}\right)$

$$\begin{aligned} f(\tau) &= \left(-\frac{du}{dr}\right) = \frac{1}{\mu}\left(\tau - \tau_0\right)^n, && \tau \ge \tau_0 \\ f(\tau) &= \left(-\frac{du}{dr}\right) = 0, && \tau \le \tau_0 \end{aligned} \tag{9}$$

$$\begin{aligned} \left(\frac{\partial u}{\partial r}\right) &= 0 && \text{at} \quad r = 0 \\ u &= 0 && \text{at} \quad r = R(z) \end{aligned} \tag{10}$$

SOLUTION OF THE PROBLEM

Solving for u from equation (8) and (9) and using boundary conditions (10), obtains,

$$u=\left(\frac{R_e\varepsilon}{2\mu}\right)^{1/n}\Bigg(r^2\frac{\left((r-R_c)^2+H^2\right)\left((r-R_c)^4+H^2\right)}{\left(1+R_c{}^2\right)^2}+\frac{\left(4R^4{}_c+H^2\right)}{\left(1+R_c{}^2\right)}r^4+\frac{\left(5R_c+H^2\right)}{\left(1+R_c{}^2\right)}r^2\Bigg)$$
$$+\frac{\left(R_e\varepsilon\frac{\partial P}{\partial Z}\right)^{(1/n)}}{(4\mu)^{(1/n)}\left(1+R_c{}^2\right)^{(1/n)}}\Bigg(r^2+\frac{\left(8R_c+H^2\right)}{\left(1+R_c{}^2\right)}r^4+\frac{\left(15(r-R_c)+H^2\right)\left(8(r-R_c)+H^2\right)}{\left(1+R_c{}^2\right)^2}\Bigg)^{(1/n)} \tag{11}$$
$$+\Bigg(\frac{H^2(r-R_c)^2}{\left(1+R^2\right)}+\frac{\left(8(r-R_c)+H^2\right)H^2}{\left(1+R^2\right)^2}\Bigg)^{2/n}\left(\frac{\partial P}{\partial Z}\right)^{(1/n)}$$

The flow rate for the blood flow with transverse magnetic field is,

$$Q=\int_0^R 2\,\pi\,u\,r\,dr \tag{12}$$

By the help of equation (11) and equation (12), flow rate can,

$$Q=\left(\frac{R_e\varepsilon}{2\mu}\left(\frac{\partial P}{\partial Z}\right)\right)^{1/n}\Bigg(\frac{(4\mu R)^{(1/n)}}{R^2}+\frac{\left(8R+H^2\right)}{(1+R)}r^4+\frac{\left(15R+H^2\right)\left(8R+H^2\right)}{R^2}+$$
$$\frac{H^2R^2}{\left(1+R^2\right)}+\frac{\left(8R+H^2\right)H^4}{\left(1+R^2\right)^2}+\frac{\left(5R+H^4\right)\left(3R+H^2\right)\left(2R+H^2\right)}{R^2}\Bigg)^{[(1/n)+1]} \tag{13}$$

From equation (13) pressure gradient is written as follows,

$$\left(\frac{\partial P}{\partial Z}\right)=\left(\frac{R_e\varepsilon}{2\mu}Q\right)^n\Bigg(\frac{(n\mu R)^n}{R^2}+\frac{\left(4n+H^2\right)\left(H^2+2n\right)}{R^2}+\frac{\left(HR_e{}^2+1\right)^n}{(2+R)^2}+$$
$$\frac{nH^2R_eR^2}{\left(1+R^2\right)}+\frac{\left(2\mu R+H^2\right)}{\left(1+R^2\right)^2}+\frac{\left(n+R^2\right)\left(3H^2+n\right)}{R^4}\Bigg)^{[(n+1]} \tag{14}$$

The dimensionless expression for resistance to flow, using

$$\lambda=\left(\frac{P_i-P_0}{Q}\right), \tag{15}$$

Using the boundary conditions, P_i is pressure at z = 0 and P_0 is the pressure at z = L. The resistance to flow can be written as:

$$\lambda=-\left(\frac{R_e\varepsilon}{n\pi}\right)^n\frac{L_0}{4\pi R_0}\frac{7\delta^2}{8R_0^2}\left(\frac{L_0}{2}\right)+$$
$$\frac{L_0}{2\pi R_0}\left[\frac{3\delta}{2R_0}+\frac{6\delta^2}{4R_0^2}+A_1\left(\frac{20\delta^2}{4R_0^2}-\frac{5\delta^2}{2R_0}\right)\right]^n$$
$$\left\{\frac{L}{R_0}\left[1-\frac{3\delta}{2R_0}+\frac{9\delta}{8R_0^2}+A_1\left(\frac{5\delta}{2R_0}-1\right)\right]^n\right.$$
$$+\frac{3\delta}{2R_0}+\frac{6\delta^2}{4R_0^2}+\frac{L_0}{4\pi R_0}\frac{7\delta^2}{8R_0^2}\left(\frac{L_0}{2}\right)$$
$$+\frac{L_0}{2\pi R_0}\left[\begin{matrix}\frac{3\delta_S}{2R_0}-\frac{6\delta^2}{4R_0^2}+\\ A_1\left(\frac{20\delta^2}{4R_0^2}-\frac{5\delta}{2R_0}\right)\end{matrix}\right]^n\left((L-d)^{(m+1)}-\frac{L_0}{2}\right)^n \tag{16}$$
$$+\left((L-d)^{(m+1)}\right)^{n+1}\frac{L_0}{2\pi R_0}\left[\begin{matrix}\frac{\delta_S}{2R_0}-\frac{2\delta^2}{5R_0^2}\\ +A_1\left(\frac{\delta^2}{3R_0^2}-\frac{5\delta}{2R_0}\right)\end{matrix}\right]^n$$

Where

$$A_1=R_e\varepsilon\left[\begin{matrix}R^2+\frac{\left(8+H^2\right)R^4}{\left(1+R^2\right)}+\\ \frac{\left(8+H^2\right)\left(15+H^2\right)R^6}{\left(1+R^2\right)^2}\end{matrix}\right]\Bigg/\left[\begin{matrix}\frac{H^2R^2}{\left(1+R^2\right)}+\\ \frac{\left(8+H^2\right)H^2R^4}{\left(1+R^2\right)^2}\end{matrix}\right]$$

The shearing stress at the wall can be written as:

$$\tau=-\mu^{3n}\left\{\begin{matrix}\left[\frac{2R_eH^2}{R^2}+\frac{4R^3\left(8+H^2\right)}{\left(1+R^2\right)^2}\right]^{3n}\left(\frac{\partial P}{\partial Z}\right)^{3n}\\ +\frac{\left(3+H^2\right)\left(2+H^2\right)}{\left(1+R^2\right)^4}\\ +\left(\frac{R_e\varepsilon\frac{\partial P}{\partial Z}}{\left(1+R^2\right)}\right)+\left[\frac{4R^3\left(8+H^2\right)}{\left(1+R^2\right)^2}\right]^{3n}\left(\frac{\partial P}{\partial Z}\right)^{3n3n}\\ +\frac{6R_e{}^2H^2(2+H^2)}{R^2}\end{matrix}\right.$$
$$\left.+\left[\frac{4R^3\left(8+H^2\right)}{\left(1+R^2\right)}+\frac{\left(8+H^2\right)\left(15+H^2\right)}{\left(1+R^2\right)^2}\right]^{3n}+\frac{4R^3\left(2+H^2\right)}{\left(2+R^2\right)^4}+\frac{\left(15+H^2\right)^2}{\left(1+R^2\right)^2}\right\} \tag{17}$$

ISSN: 0976 - 2965
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RESULTS AND DISCUSSION

In order to have estimate of the quantitative effects of various parameters such as Hartmann number (H= 0, 0.2, 0.6, 1), red cell (M = 0, 2, 4), stenosis shape parameter (m= 2...11), flow behavior index (n= 1, 2/3, 1/3) involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, velocity profile and wall shear stress for diseased system associated with stenosis due to the local deposition of lipids have been determine. The results are shown in Fig 2-6 by using the values of parameter based on experimental data in stenosed artery.

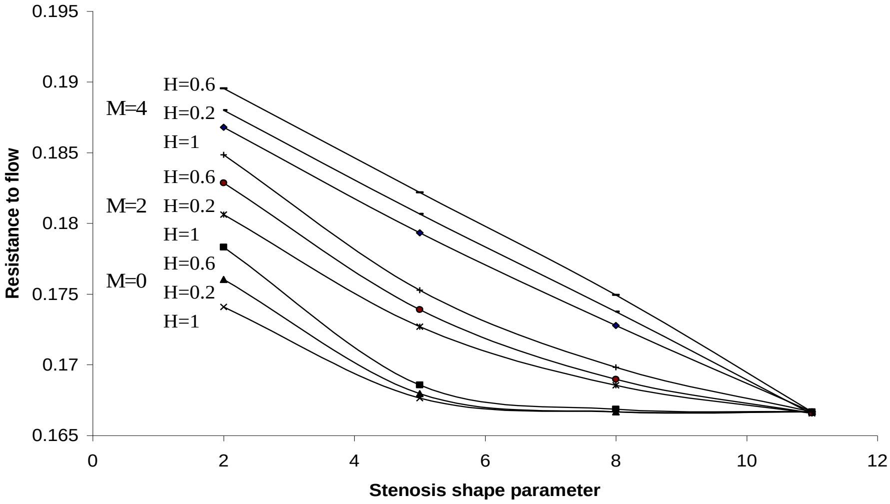

Fig. (2) Variation of resistance to flow with stenosis shape parameter for different H

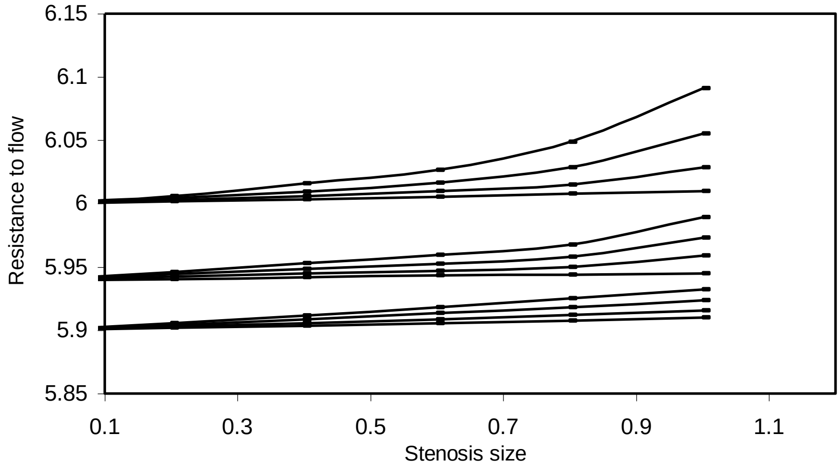

Fig (3) Variation of resistance to flow with stenosis size for different H

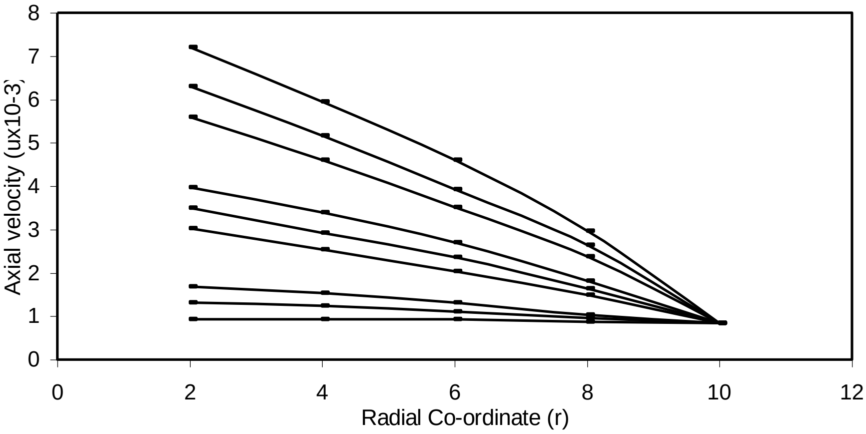

Fig (4). Variation of axial velocity with radial co-ordinate

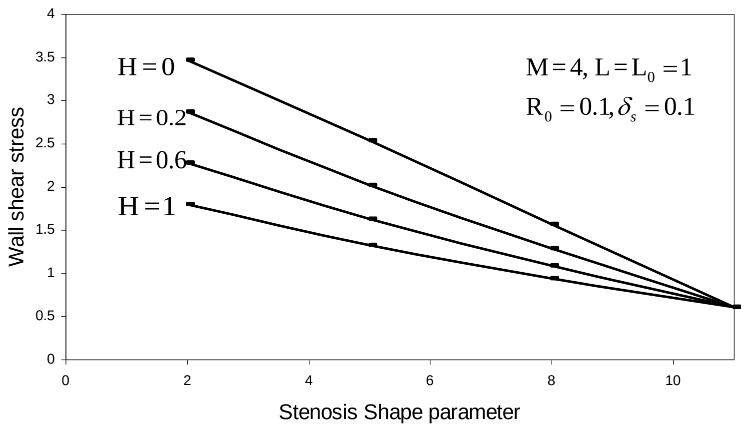

Fig (5). Variation of wall shear stress with m

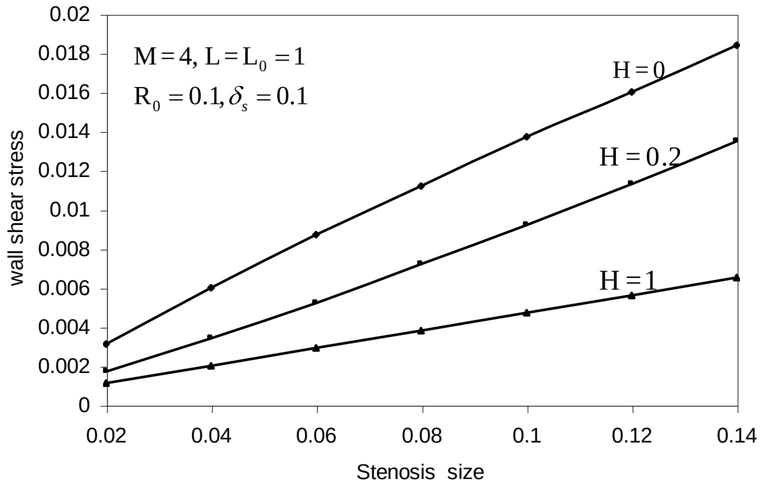

Fig. (6) Variation of wall shear stress with stenosis size

The variation of resistance to flow (λ) with stenosis shape parameter (m) for different values of Hartmann number (H) and red cell parameter (M) are shown in Fig. 2. It is evident that resistance to flow decreases as stenosis shape parameter increases. Increase in the resistance to flow with red cell is also indicated in the Fig. 2. It is also seen from the figure maximum resistance to flow occurs at m = 2. i e. in the case of symmetric stenosis. Resistance to flow increase as stenosis grows or radius of artery decreases (this referred to as Fahraeus-Lindquist effect in very thin tubes). It is also shown in figure that the resistance to flow increases with decreasing value of Hartmann number. Mazumdar [10] found that the resistance of blood in diabetic patients is higher than in non-diabetic patients, resulting higher resistance to blood flow in the presence of magnetic effect. Thus diabetic patients with higher resistance to flow are more prone to high blood pressure. Therefore the resistance to blood flow in case of diabetic patients may be reduced by reducing by reducing viscosity of the plasma. These results are consistent with the observation of [3, 5] viscosity of blood in stenotic region, therefore increases. Our results are similar to [13]. Therefore magnetic field can be effectively utilized to deaccelerate the blood flow in flow problems.

Fig.3 shows the results for resistance to flow for different values of stenosis size, Hartmann number and red cell parameter. Resistance to flow increases as stenosis size increases and decreases as Hartmann numbers (H) increases. It is also shown in figure that the resistance to flow increases with increasing value of red cell parameter (M). Fig. 4 depicts the variation of axial velocity with radial co-ordinate. It is clear from the figure that of magnetic field reduces the velocity of blood. The development of stenosis accelerates the velocity of plasma between the cells. This in turn increases the concentration of red cell and

In a healthy artery, the wall shear stress is approximately 15 dyn / cm^2. The interior surface will be damaged once the wall shear stress reaches a value higher than 400 dyn / cm^2 [7]. Therefore to determine the critical flow condition, prediction of wall shear stress using numerical experiments become necessary. To capture the results for wall shear stress in the presence of externally applied magnetic field, the

graphs have been plotted in figure 5 and figure 6. These graphs show the results for wall shear stress (τ) for different values of stenosis shape parameter (m), stenosis length, Hartmann numbers (H), red cell (M) and stenosis size (δ/R_0). Wall shear stress (τ) increases as stenosis size increases and decreases as stenosis shape parameter (m) and Hartmann number (H) increases. It is concluding from these results that the wall shear stress (τ) increases as the stenosis grows and remains constants outside from the stenotic region. It is also seen that wall shear stress is decreases when an external magnetic field is applied. This result is qualitative agreement with the observation of [8, 11]. It is observed that reduction in velocity, resistances to flow and wall shear stress are more pronounced in central region of the pipe and results are compared with [10].

CONCLUSIONS

Blood flow through an artery mainly depends on the pressure gradient and resistance to flow. Resistance to flow increases as the stenosis grows and remains constant outside the stenotic region. If resistance to flow increases, it is more difficult for the blood to pass through an artery, result the flow decreases and heart has to work harder to maintain adequate circulation. In this present study, velocity profile, resistance to flow and wall shear stress are obtained when blood is assuming as Herschel-Bulkely electrically conducting fluid, so that the effect of magnetic field on blood flow through an artery can be observed. Looking at the importance of the hydrodynamic factors in the understanding of blood flow and atherosclerotic diseases, it may be said that the present model could be useful for investigating blood flow through stenosed artery, in particular in diseased stage when blood is no longer a Newtonian fluid, it has stress with magnetic effects. It is noticed that the resistance to blood flow and wall shear stress decreases for different magnetic fields and also decreases as stenosis shape parameter increases. The magnetic field is to decrease the resistance to flow due to irregular boundaries. So the magnetic field can be effectively utilized to deaccelerate the blood flow in flow problems. The present study is able to predict the main characteristic of the physiological flows and may have played some important role in biomedical application.

REFERENCES

[1] Young DF: Effect of a time-dependent stenosis of flow through a tube, J. Eng. Ind. (1968) 248-254.

[2] Nerem RM: Fluid dynamics aspects of arterial disease, In proceeding of a specialist meeting at Ohio University held in sept. (1974) 19-20.

[3] Haldar K: Effect of the shape of stenosis on the resistance to flow through an artery, Bull. Math. Bio. 47 (1985) 545-550.

[4] Bitoun JP and Bellet D: Blood flow through a stenosis in microcirculation, Biorheol. 23 (1986) 51-61.

[5] Chakravarty S: Effect of stenosis on flow behaviour of blood in an artery, Int. J. Eng. Sci. 25 (1987) 1003-1016.

[6] Srivastava VP and Saxena M: Two-layered model of Casson's fluid flow through stenotic blood vessels: application to cardiovascular system, J Biomech. 27 (1994) 921-928.

[7] Srivastava LM: Flow of couple stress fluid through stenotic blood vessels, J Biomech. 18 (1985) 479-485.

[8] Srivastava VP: Particulate suspension blood flow through stenotic arteries: Effect of Hematocrit and stenosis shape, I. J. Pure Appl. Math. 33(9) (2002) 1353-1360.

[9] Mazumdar HP, Ganguly UN and Venkatesan SK: Some effects of magnetic field on flow of a Newtonian fluid through a circular tube. Indian J Pure and Appl. Math. 27 (5) (1996) 519-524.

[10] Tandon PN and Pal TS: Applied magnetic field effects on pulsatile blood through a time developing stenotic tube, Proc 16th national Conf. Fluid Mech. and fluid power. (1988).

[11] Sud VK, Suri PK and Mishra RK: Effect of magnetic field on oscillating blood in arteries, Studia Biophysica, 46 (1974)163-169.

[12] Mazumdar HP Ganguly UN and Venkatesan SK: Some effects of magnetic field on the flow of a Newtonian fluid through a circular tube, Indian J. Pure and Appl. Math.27 (5) (1996).

[13] Barnothy MF: Biological Effects of Magnetic Fields, (1,2) Plenum Press, (1969).

[14] Sud VK and Sekhon GS: Blood Flow through the Human Arterial system in the presence of a steady magnetic field, Phys. Med. Biol, 34(1989) 36-42.

[15] Tzirtzilakis EE: A mathematical model for blood flow in magnetic field, Phys. Fluids 103 (2005) 77-103.

[16] Amos E and Ogulu A: Magnetic effect on pulsatile flow in a constricted axissymmetric tube. Indian J. Pure and appl. Math. 34(9) (2003) 1315-1326.

[17] Haldar K and Ghosh SN: Effect of Magnetic field on blood flow through an indented tube in the presence of eeythrocytes, Indian. J. Pure appl. Math. 25(3) (1994) 345-352.

[18] Secomb TW Pries. AR, Blood flow and red blood cell deformation in non-uniform capillaries: effect of endothelial surface layer. Microc. 9, (2005) 189-196

[19] Vittorio C and Ghassan, SK: Computer Modeling of red blood cell Rheology in the Microcirculation: A Brief Overview. Life Science, 33 (2008) 1-4.